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On the Influence of Gravity on the Surface Rayleigh and Love Waves treated in Cylindrical Co-ordinates.

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On the Influence of Gravity on Surface (Rayleigh and Love) Waves treated in Cylindrical Co-ordinates.

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The influences of gravity on simple Rayleigh waves were first treated by Bromwich,[1] assuming the material is incompressible in order to avoid the mathematical difficulties. Following the usual method, he made one modification, viz., the normal traction on the mean free surface has to be sufficient to support the weight of the harmonic inequality, instead of vanishing. In this paper, the problem is treated in different ways, using the usual method for an incompressible fluid under the influence of gravity, in hydrodynamics; and the present author intends to discuss the influence of gravity on surface waves in cylindrical co-ordinates.

1. We shall take the origin on the free surface of the earth and draw the axis of z vertically upwards. Then the equations of motion of a homogeneous and isotropic elastic solid in cylindrical coordinates (r, θ, z) are given by,

$$\left.\begin{aligned}
\rho\frac{\partial^2\vartheta_r}{\partial t^2}&=(\lambda+\mu)\frac{\partial\Delta}{\partial r}+\mu\left\{\frac{\partial^2\vartheta_r}{\partial r^2}+\frac{1}{r}\frac{\partial\vartheta_r}{\partial r}+\frac{\partial^2\vartheta_r}{\partial z^2}-\frac{1}{r^2}\left(\vartheta_r-\frac{\partial^2\vartheta_r}{\partial\theta^2}\right)-\frac{2}{r^2}\frac{\partial\vartheta_\theta}{\partial\theta}\right\}\\
\rho\frac{\partial^2\vartheta_\theta}{\partial t^2}&=(\lambda+\mu)\frac{1}{r}\frac{\partial\Delta}{\partial\theta}+\mu\left\{\frac{\partial^2\vartheta_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\vartheta_\theta}{\partial r}+\frac{\partial^2\vartheta_\theta}{\partial z^2}-\frac{1}{r^2}\left(\vartheta_\theta-\frac{\partial^2\vartheta_\theta}{\partial\theta^2}\right)+\frac{2}{r^2}\frac{\partial\vartheta_r}{\partial\theta}\right\}\\
\rho\frac{\partial^2\vartheta_z}{\partial t^2}&=-\rho g+(\lambda+\mu)\frac{\partial\Delta}{\partial z}+\mu\left\{\frac{\partial^2\vartheta_z}{\partial r^2}+\frac{1}{r}\frac{\partial\vartheta_z}{\partial r}+\frac{\partial^2\vartheta_z}{\partial z^2}+\frac{1}{r^2}\frac{\partial^2\vartheta_z}{\partial\theta^2}\right\}
\end{aligned}\right\},$$

where $\vartheta(\vartheta_r, \vartheta_\theta, \vartheta_z)$ denotes the displacement
t the time
ρ the density
λ, μ Lamé's constants

and
$$\Delta=\operatorname{div}\vartheta=\frac{\partial}{\partial r}(r\vartheta_r)+\frac{1}{r}\frac{\partial\vartheta_\theta}{\partial\theta}+\frac{\partial\vartheta_z}{\partial z}.$$

We shall assume the material is incompressible in order to avoid the

(1) T. J. I'A. Bromwich: On the Influence of Gravity on Elastic Waves, and in Particular, on the Vibration of an Elastic Glove, Proc. London Math. Soc., Vol. XXX, 1898.

difficulties that arise, following Bromwich and Lamb's method. Since the solid is incompressible, Δ, the dilatation, will be zero, however $\lambda\Delta$ will be finite and let us put $\Pi=-\lambda\Delta$, so that Π denotes a hydrostatic pressure.

We see that if we put

$$\left.\begin{aligned} \vartheta_r&=\frac{\partial\varphi}{\partial r}-\frac{\partial A_\theta}{\partial z}\\ \vartheta_\theta&=-\frac{\partial A_z}{\partial r}\\ \vartheta_z&=\frac{\partial\varphi}{\partial z}+\frac{\partial A_\theta}{\partial r}+\frac{A_\theta}{r} \end{aligned}\right\}^{*}, \tag{1}$$

the equations of motion are satisfied, provided

$$\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}+\frac{\partial^2\varphi}{\partial z^2}=0, \tag{2}$$

$$\left.\begin{aligned} \rho\frac{\partial^2 A_\theta}{\partial t^2}&=\mu\left(\frac{\partial^2 A_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial A_\theta}{\partial r}+\frac{\partial^2 A_\theta}{\partial z^2}-\frac{A_\theta}{r^2}\right)\\ \rho\frac{\partial^2 A_z}{\partial t^2}&=\mu\left(\frac{\partial^2 A_z}{\partial r^2}+\frac{1}{r}\frac{\partial A_z}{\partial r}+\frac{\partial^2 A_z}{\partial z^2}\right) \end{aligned}\right\}, \tag{3}$$

and

$$\Pi=-\rho\frac{\partial^2\varphi}{\partial t^2}-\rho gz.^{(1)} \tag{4}$$

The solutions of the equations (2) and (3) are of the form

$$\left.\begin{aligned} \varphi&=Pe^{\xi z}C_0(\xi r)e^{ipt}\\ A_\theta&=Re^{sz}C_1(\xi r)e^{ipt}\\ A_z&=Se^{sz}C_0(\xi r)e^{ipt} \end{aligned}\right\}, \tag{5}$$

where P, R, S are constants and $C_0(\xi r)$, $C_1(\xi r)$ the cylinder functions of the order 0 and 1, respectively, and

$$s^2=\xi^2-k^2,\quad k^2=\frac{\rho p^2}{\mu}.$$

Then, by (1), we have

$$\left.\begin{aligned} \vartheta_r&=-(\xi Pe^{\xi z}+sRe^{sz})C_1(\xi r)e^{ipt}\\ \vartheta_\theta&=\xi Se^{sz}C_1(\xi r)e^{ipt}\\ \vartheta_z&=\xi(Pe^{\xi z}+Re^{sz})C_0(\xi r)e^{ipt} \end{aligned}\right\}. \tag{6}$$

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* We denote by φ and $\mathfrak{A}$ the scalor and vector potential, respectively, and assume the motion be symmetrical about the axis of z, i.e., $\frac{\partial}{\partial\theta}\equiv 0$.

(1) Bromwich treated the problem in different ways, for he used the equation of motion for no external force and considered the effect of gravity only for the tractions at the free surface. Here the author applied the method used for an incompressible fluid in hydrodynamics.

Since

$$\left.\begin{aligned}\widehat{zr}&=\mu\left(\frac{\partial\vartheta_r}{\partial z}+\frac{\partial\vartheta_z}{\partial r}\right)\\ \widehat{z\theta}&=\mu\left(\frac{1}{r}\frac{\partial\vartheta_z}{\partial\theta}+\frac{\partial\vartheta_\theta}{\partial z}\right)\\ \widehat{zz}&=-\Pi+2\mu\frac{\partial\vartheta_z}{\partial z}\end{aligned}\right\},$$

the corresponding stress components are given by

$$\left.\begin{aligned}\frac{\widehat{zr}}{\mu}&=-\{2\xi^2Pe^{\xi z}+(2\xi^2-k^2)Re^{sz}\}C_1(\xi r)e^{ipt}\\ \frac{\widehat{z\theta}}{\mu}&=\xi sSe^{sz}C_1(\xi r)e^{ipt}\\ \frac{\widehat{zz}}{\mu}&=\{(2\xi^2-k^2)Pe^{\xi z}+2\xi sRe^{sz}\}C_0(\xi r)e^{ipt}+\frac{\rho}{\mu}gz\end{aligned}\right\}. \tag{7}$$

Since the elevation ξ at the free surface is

$$\eta=\xi(P+R)C_0(\xi r)e^{ipt}, \tag{8}$$

the condition, the normal stresses must vanish at the free surface, is

$$\left\{(2\xi^2-k^2)+\xi\frac{\rho g}{\mu}\right\}P+\left(2\xi s+\xi\frac{\rho g}{\mu}\right)R=0. \tag{9}$$

The conditions, the tangential stresses must vanish at the free surface, are

$$2\xi^2P+(2\xi^2-k^2)R=0, \tag{10}$$

$$S=0. \tag{11}$$

Elliminating P and R among (9) and (10), we have

$$(2\xi^2-k^2)^2-4\xi^3s-\xi k^2\frac{\rho g}{\mu}=0. \tag{12}$$

If we put $\zeta=\left(\frac{k}{\xi}\right)^2$, then we have

$$(2-\zeta)^2-4\sqrt{1-\zeta}-\zeta\frac{\rho g}{\mu\xi}=0, \tag{12-a}$$

which is the same equation obtained by Bromwich.[1] We denote by κ a real positive root of (12). The corresponding values of s are then

$$s_\kappa=\sqrt{\kappa^2-k^2}.$$

Now the solutions are

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(1) loc. cit., p. 102.

$$\left.\begin{aligned}
\vartheta_r &= A\kappa\{(2\kappa^2-k^2)e^{\kappa z}-2\kappa s_\kappa e^{s_\kappa z}\}C_1(\kappa r)e^{ipt}\\
\vartheta_\theta &= 0\\
\vartheta_z &= -A\kappa\{(2\kappa^2-k^2)e^{\kappa z}-2\kappa^2 e^{s_\kappa z}\}C_0(\kappa r)e^{ipt}
\end{aligned}\right\},$$

where

$$P : R : A = -(2\kappa^2-k^2) : 2\kappa^2 : 1.$$

If we put $H_{2,0}(\kappa r)$ and $H_{2,1}(\kappa r)$ in the places of $C_0(\kappa r)$ and $C_1(\kappa r)$, respectively, in these expressions, we get the wave diverging $r=\infty$, when κr is large. Putting

$$C_0(\kappa r)=H_{2,0}(\kappa r),$$
$$C_1(\kappa r)=H_{2,1}(\kappa r),$$

we obtain

$$\left.\begin{aligned}
\vartheta_r &= A\kappa\{(2\kappa^2-k^2)e^{\kappa z}-2\kappa s_\kappa e^{s_\kappa z}\}H_{2,1}(\kappa r)e^{ipt}\\
\vartheta_\theta &= 0\\
\vartheta_z &= -A\kappa\{(2\kappa^2-k^2)e^{\kappa z}-2\kappa^2 e^{s_\kappa z}\}H_{2,0}(\kappa r)e^{ipt}
\end{aligned}\right\}. \tag{13}$$

When κr is large, we can use the asymtotic expansion

$$H_{2,n}(\kappa r)=\sqrt{\frac{2}{\pi\kappa r}}\,e^{-i\left(\kappa r-\frac{2n+1}{4}\pi\right)}\{P_\nu(\kappa r)-iQ_\nu(\kappa r)\},$$

where

$$\left.\begin{aligned}
P_\nu(\kappa r) &= 1+\sum_{j=1}^{j\leq\frac{\nu}{2}}\frac{(-1)^j\left(n^2-\frac{1}{4}\right)\left(n^2-\frac{3^2}{4}\right)\cdots\left(n^2-\frac{(4j-1)^2}{4}\right)}{(2j)!\,(2\kappa r)^{2j}}\\
Q_\nu(\kappa r) &= \sum_{j=1}^{j\leq\frac{\nu-1}{2}}\frac{(-1)^j\left(n^2-\frac{1}{4}\right)\left(n^2-\frac{3^2}{4}\right)\cdots\left(n^2-\frac{(4j+1)^2}{4}\right)}{(2j+1)!\,(2\kappa r)^{2j+1}}
\end{aligned}\right\}.$$

For very large values of κr, we have

$$P_\nu(\kappa r)=1,\qquad Q_\nu(\kappa r)=0,$$

and so

$$H_{2,n}(\kappa r)=\sqrt{\frac{2}{\pi\kappa r}}\,e^{-i\left(\kappa r-\frac{2n+1}{4}\pi\right)},$$

and

$$\left.\begin{aligned}
\vartheta_r &= A\kappa\{(2\kappa^2-k^2)e^{\kappa z}-2\kappa s_\kappa e^{s_\kappa z}\}\sqrt{\frac{2}{\pi\kappa r}}\,e^{i\left(pt-\kappa r+\frac{3}{4}\pi\right)}\\
\vartheta_\theta &= 0\\
\vartheta_z &= -A\kappa\{(2\kappa^2-k^2)e^{\kappa z}-2\kappa^2 e^{s_\kappa z}\}\sqrt{\frac{2}{\pi\kappa r}}\,e^{i\left(pt-\kappa r+\frac{1}{4}\pi\right)}
\end{aligned}\right\}. \tag{14}$$

Thus the amplitudes diminish inversely square root of r, and the wave length L and the velocity V are given by

$$L=\frac{2\pi}{\kappa}, \tag{15}$$

$$V=\frac{p}{\kappa}. \tag{16}$$

Now (12) is the equation for determining the velocity of Rayleigh waves. As Bromwith discussed, we can see that the influence of gravity on the Rayleigh waves is almost insensible.

2. To discuss the surface waves in two layer earth crust, we shall take the origin on the under-surface of the superficial layer and draw the axis of z vertically upwards.

We denote the density and elastic constants of the upper medium by ρ_1, λ_1, μ_1, and those of the lower medium by ρ_2, λ_2, μ_2; the corresponding ϑ, II, etc., will be distinguished by adding indices 1, 2, respectively.

The solutions of the equations (2) and (3) are of the form

$$\left.\begin{aligned}\varphi_1&=(P_1\cosh\xi z+P_1'\sinh\xi z)C_0(\xi r)e^{ipt}\\ A_{\theta_1}&=(R_1\cosh s_1z+R_1'\sinh s_1z)C_1(\xi r)e^{ipt}\\ A_{z_1}&=(S_1\cosh s_1z+S_1'\sinh s_1z)C_0(\xi r)e^{ipt}\end{aligned}\right\}, \tag{17}$$

where $P_1, P_1', R_1, R_1', S_1, S_1'$ are constants, $C_0(\xi r)$ and $C_1(\xi r)$ any cylinder functions of the order 0 and 1, respectively, and

$$s_1^2=\xi^2-k_1^2, \qquad k_1^2=\frac{\rho_1p^2}{\mu_1},$$

for the upper medium. For the subjacent material, we have

$$\left.\begin{aligned}\varphi_2&=P_2e^{\xi z}C_0(\xi r)e^{ipt}\\ A_{\theta_2}&=R_2e^{s_2z}C_1(\xi r)e^{ipt}\\ A_{z_2}&=S_2e^{s_2z}C_0(\xi r)e^{ipt}\end{aligned}\right\}, \tag{18}$$

where P_2, R_2, S_2 are constants and

$$s_2^2=\xi^2-k_2^2, \quad k_2^2=\frac{\rho_2p^2}{\mu_2}.$$

Then, by (1), we have

$$\left.\begin{aligned}\vartheta_{r_1}&=-\{\xi(P_1\cosh\xi z+P_1'\sinh\xi z)+s_1(R_1\sinh s_1z+R_1'\cosh s_1z)\}C_1(\xi r)e^{ipt}\\ \vartheta_{\theta_1}&=\xi(S_1\cosh s_1z+S_1'\sinh s_1z)C_1(\xi r)e^{ipt}\\ \vartheta_{z_1}&=\xi(P_1\sinh\xi z+P_1'\cosh\xi z+R_1\cosh s_1z+R_1'\sinh s_1z)C_0(\xi r)e^{ipt}\end{aligned}\right\}, \tag{19}$$

for the upper medium and

$$\left.\begin{aligned}\vartheta_{r_2}&=-(\xi P_2e^{\xi z}+s_2R_2e^{s_2z})C_1(\xi r)e^{ipt}\\ \vartheta_{\theta_2}&=\xi S_2e^{s_2z}\cdot C_1(\xi r)\cdot e^{ipt}\\ \vartheta_{z_2}&=\xi(P_2e^{\xi z}+R_2e^{s_2z})\cdot C_0(\xi r)e^{ipt}\end{aligned}\right\}, \tag{20}$$

for the lower medium. The corresponding stress components are given by

$$\left.\begin{aligned}
\frac{\widehat{zr_1}}{\mu_1} &= -\{2\xi^2(P_1\sinh\xi z + P_1'\cosh\xi z) + (2\xi^2 - k_1^2)(R_1\cosh s_1 z + R_1'\sinh s_1 z)\} \\
&\qquad\qquad\times C_1(\xi r)e^{ipt} \\
\frac{\widehat{z\theta_1}}{\mu_1} &= \xi S_1(S_1\sinh s_1 z + S_1'\cosh s_1 z)C_1(\xi r)e^{ipt} \\
\frac{\widehat{zz_1}}{\mu_1} &= \{(2\xi^2 - k_1^2)(P_1\cosh\xi z + P_1'\sinh\xi z) + 2\xi s_1(R_1\sinh s_1 z + R_1'\cosh s_1 z)\} \\
&\qquad\qquad\times C_0(\xi r)e^{ipt} + \frac{\rho_1}{\mu_1}gz
\end{aligned}\right\}, \tag{21}$$

for the upper medium and

$$\left.\begin{aligned}
\frac{\widehat{zr_2}}{\mu_2} &= -\{2\xi^2 P_2 e^{\xi z} + (2\xi^2 - k_2^2)R_2 e^{s_2 z}\}C_1(\xi r)e^{ipt} \\
\frac{\widehat{z\theta_2}}{\mu_2} &= \xi s_2 S_2 e^{s_2 z}C_1(\xi r)e^{ipt} \\
\frac{\widehat{zz_2}}{\mu_2} &= \{(2\xi^2 - k_2^2)P_2 e^{\xi z} + 2\xi s_2 R_2 e^{s_2 z}\}C_0(\xi r)e^{ipt} + \frac{\rho_2}{\mu_2}gz
\end{aligned}\right\}, \tag{22}$$

for the lower medium.

The conditions of continuity of displacement at the lower boundary of the layer are

$$\left.\begin{aligned}
\xi P_1 + s_1 R_1' &= \xi P_2 + s_2 R_2 \\
S_1 &= S_2 \\
P_1' + R_1 &= P_2 + R_2
\end{aligned}\right\}. \tag{23}$$

The conditions of continuity of stress at the lower boundary of the layer are

$$\left.\begin{aligned}
&\mu_1\{2\xi^2 P_1' + (2\xi^2 - k_1^2)R_1\} = \mu_2\{2\xi^2 P_2 + (2\xi^2 - k_2^2)R_2\} \\
&\mu_1 s_1 S_1' = \mu_2 s_2 S_2 \\
&\mu_1\{(2\xi^2 - k_1^2)P_1 + 2\xi s_1 R_1'\} + \rho_1 g\xi(P_1' + R_1) \\
&\qquad\qquad = \mu_2\{(2\xi^2 - k_2^2)P_2 + 2\xi s_2 R_2\} + \rho_2 g\xi(P_2 + R_2)
\end{aligned}\right\}. \tag{24}$$

The conditions that the plane $z=h$ may be free from traction are

$$\left.\begin{aligned}
&2\xi^2(P_1\sinh\xi h + P_1'\cosh\xi h) + (2\xi^2 - k_1^2)(R_1\cosh s_1 h + R_1'\sinh s_1 h) = 0 \\
&S_1\sinh s_1 h + S_1'\cosh s_1 h = 0 \\
&(2\xi^2 - k_1^2)(P_1\cosh\xi h + P_1'\sinh\xi h) + 2\xi s_1(R_1\sinh s_1 h + R_1'\cosh s_1 h) \\
&\qquad + \frac{\rho_1}{\mu_1}g\xi(P_1\sinh\xi h + P_1'\cosh\xi h + R_1\cosh s_1 h + R_1'\sinh s_1 h) = 0
\end{aligned}\right\}. \tag{25}$$

3. *Love Waves.* From the three equations of (23.2), (24.2) and (25.2), we have

$$S_1 = S_2$$
$$\mu_1 s_1 S_1 = \mu_2 s_2 S_2$$
$$S_1 \sinh s_1 h + S_1' \cosh s_1 h = 0.$$

Elliminating S_1, S_1' and S_2 among these equations, we get

$$\mu_2 s_2 \cosh s_1 h + \mu_1 s_1 \sinh s_1 h = 0. \tag{26}$$

We confine ourselves to a real value of ξ and to the case

$$k_1 > k_2, \quad \text{i.e.,} \quad \frac{\mu_1}{\rho_1} < \frac{\mu_2}{\rho_2},$$

the velocity of the distortional waves of the upper medium is smaller than that of the lower medium.

Then the roots of (26) are, putting $s_1 = i\bar{s_1}$,

$$\tan \bar{s_1} f = \frac{\mu_2 s_2}{\mu_1 \bar{s_1}},$$

which is identical with that given by Love,[1] except the difference of notations

Hence no influence of gravity on Love waves exists, so long as the waves is symmetric with respect to the axis of z. As for more details on Love waves, the author must refer to readers other papers.[2]

4. *Rayleigh Waves in Two Layer Crust.* From the six equations of (23.1), (23.3), (24.1), (24.3), (25.1) and (25.3), we have

$$\xi P_1 + s_1 R_1' = \xi P_2 + s_2 R_2$$
$$P_1' + R_1 = P_2 + R_2$$
$$\mu_1 \{2\xi^2 P_1' + (2\xi^2 - k_1^2) R_1\} = \mu_2 \{2\xi^2 P_2 + (2\xi^2 - k_2^2) R_2\}$$
$$\mu_1 \{(2\xi^2 - k_1^2) P_1 + 2\xi s_1 R_1'\} + \rho_1 g \xi (P_1' + R_1)$$
$$= \mu_2 \{(2\xi^2 - k_2^2) P_2 + 2\xi s_2 R_2\} + \rho_2 g \xi (P_2 + R_2)$$
$$2\xi^2 (P_1 \sinh \xi h + P_1' \cosh \xi h) + (2\xi^2 - k_1^2)(R_1 \cosh s_1 h + R_1' \sinh s_1 h) = 0$$
$$(2\xi^2 - k_1^2)(P_1 \cosh \xi h + P_1' \sinh \xi h) + 2\xi s_1 (R_1 \cosh s_1 h + R_1' \sinh s_1 h)$$
$$+ \frac{\rho_1 g}{\mu_1} \xi (P_1 \sinh \xi h + P_1' \cosh \xi h + R_1 \cosh s_1 h + R_1' \sinh s_1 h) = 0.$$

If we put $g = 0$, we get the six equations given by Love,[3] except the difference of the notations. Elliminating $P_1, P_1', R_1, R_1', P_2$ and R_2 among these equations, we get

(1) Love: Geodynamics, p. 162, Eq. (40).

(2) H. Nakano: Love Waves in Cylindrical Co-ordinates, Geophys. Mag., Vol. II, p. 37 (1928); H. Arakawa: Surface Waves in Two Layer Crust, Geophys. Mag., Vol. V, p. 123 (1932).

(3) Geodynamics, pp. 167-8. The constants. P_1, P_1', R_1, R_1'; P_2, R_2; ξ, k_1, k_2 correspond to P, Q, $-A$, $-B$; P', $-A'$; f, κ, κ'; respectively.

$$\begin{vmatrix} \xi & 0 & 0 & s_1 & \xi & s_2 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 2\xi^2\mu_1 & (2\xi^2-k_1^2)\mu_1 & 0 & 2\xi^2\mu_2 & (2\xi^2-k_2^2)\mu_2 \\ (2\xi^2-k_1^2)\mu_1 & \rho_1 g\xi & \rho_1 g\xi & 2\xi s_1\mu_1 & (2\xi^2-k_2^2)\mu_2+\rho_2 g\xi & 2\xi s_2\mu_2+\rho_2 g\xi \\ 2\xi^2\sinh\xi h & 2\xi^2\cosh\xi h & (2\xi^2-k_1^2)\cosh s_1 h & (2\xi^2-k_1^2)\sinh s_1 h & 0 & 0 \\ \begin{matrix}(2\xi^2-k_1^2)\cosh\xi h \\ +\frac{\rho_1 g}{\mu_1}\xi\sinh\xi h\end{matrix} & \begin{matrix}(2\xi^2-k_1^2)\sinh\xi h \\ +\frac{\rho_1 g}{\mu_1}\xi\cosh\xi h\end{matrix} & \begin{matrix}2\xi s_1\sinh s_1 h \\ +\frac{\rho_1 g}{\mu_1}\xi\cosh s_1 h\end{matrix} & \begin{matrix}2\xi_1 s_1\cosh s_1 h \\ +\frac{\rho_1 g}{\mu_1}\xi\sinh s_1 h\end{matrix} & 0 & 0 \end{vmatrix} = 0,$$

which is the equation for determining the velocity of Rayleigh waves in terms of the wave length.

Again as Bromwich has shown,

$$\frac{\rho_1}{\mu_1} g \approx 10^{-8}\ C.G.S.,$$

roughly. Hence we can neglect the terms including the acceleration of gravity, and we get the equation for determining the velocity of Rayleigh waves in terms of the wave length as

$$\begin{vmatrix} \xi & 0 & 0 & s_1 & \xi & s_2 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 2\xi^2\mu_1 & (2\xi^2-k_1^2)\mu_1 & 0 & 2\xi^2\mu_2 & (2\xi^2-k_2^2)\mu_2 \\ (2\xi^2-k_1^2)\mu_1 & 0 & 0 & 2\xi s_1\mu_1 & (2\xi-k_2^2)\mu_2 & 2\xi s_2\mu_2 \\ 2\xi^2\sinh\xi h & 2\xi^2\cosh\xi h & (2\xi^2-k_1^2)\cosh s_1 h & (2\xi^2-k_1^2)\sinh s_1 h & 0 & 0 \\ (2\xi^2-k_1^2)\cosh\xi h & (2\xi^2-k_1^2)\sinh\xi h & 2\xi s_1\sinh s_1 h & 2\xi s_1\cosh s_1 h & 0 & 0 \end{vmatrix} = 0.$$

This equation was solved approximately by Love. Hence the velocity of Rayleigh waves in two layer crust treated in cylindrical coordinates is identical with that of Rayleigh waves in two layer, treated as two dimensional.

5. *Conclusion.* Using the cylindrical co-ordinates, the effect of gravity is studied, under the simplifying assumption of incompressiblity in order to avoid the mathematical difficulties.

The effect of gravity on simple Rayleigh waves is insensibly small as Bromwich studied.

On Love waves, there exists no influence of gravity.

Again the influence of gravity on Rayleigh waves in two layer crust is also insensibly small.

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